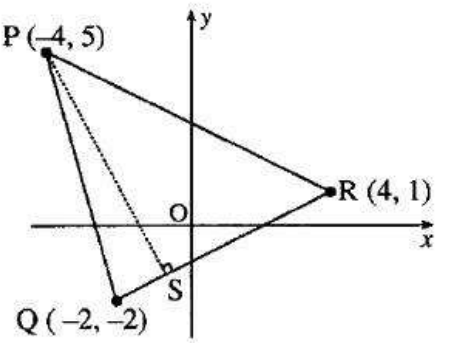
perpendicular gradients

[SQA] 1. Find the equation of the perpendicular bisector of the line joining $A(2,-1)$ and $B(8,3)$. **4**

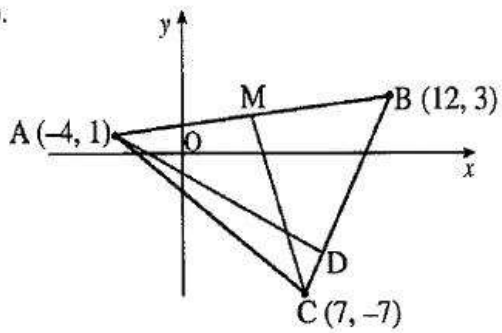
[SQA] 2. P(–4,5), Q(–2,–2) and R(4, 1) are the vertices of triangle PQR as shown in the diagram. Find the equation of PS, the altitude from P. **3**



[SQA] 3. The vertices of a triangle are $P(-1,1)$, $Q(2,1)$ and $R(-6,2)$. Find the equation of the altitude of triangle PQR, drawn from P. **3**

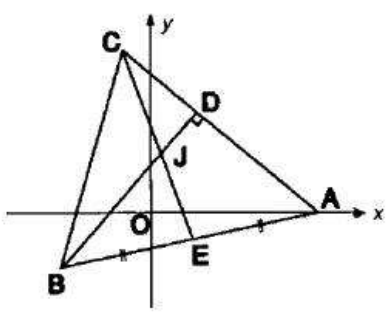
[SQA] 4. A triangle ABC has vertices A (–4, 1), B (12, 3) and C (7, –7).

(*a*) Find the equation of the median CM. **3**
(*b*) Find the equation of the altitude AD. **3**
(*c*) Find the coordinates of the point of intersection of CM and AD. **3**

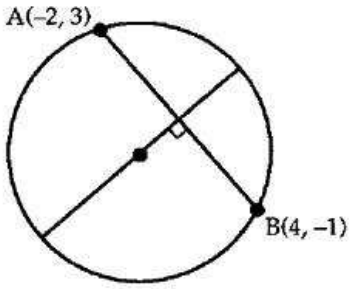


[SQA] 5. In the diagram A is the point (7,0), B is (–3,– 2) and C(–1,8). The median CE and the altitude BD intersect at J.

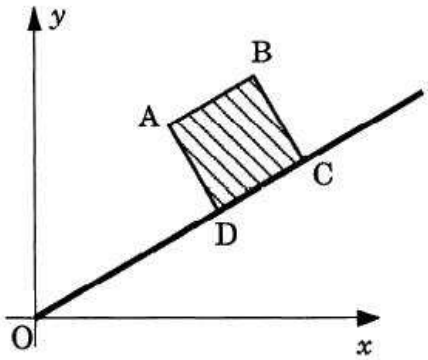
(a) Find the equations of CE and BD. **6**
(b) Find the co-ordinates of J. **2**



[SQA] 6. A circle passes through A(–2, 3) and B(4, –1). Find the equation of the perpendicular to the chord AB. 4



[SQA] 7. ABCD is a square. A is the point with coordinates (3,4) and ODC has equation $y = \frac{1}{2}x$.



(*a*) Find the equation of the line AD. **(3)**

(*b*) Find the coordinates of D. **(3)**

(*c*) Find the area of the square ABCD. **(2)**

[SQA] 8. P, Q and R have coordinates $(1, -2)$, $(6, 3)$ and $(9, 14)$ respectively and are three vertices of a kite PQRS.

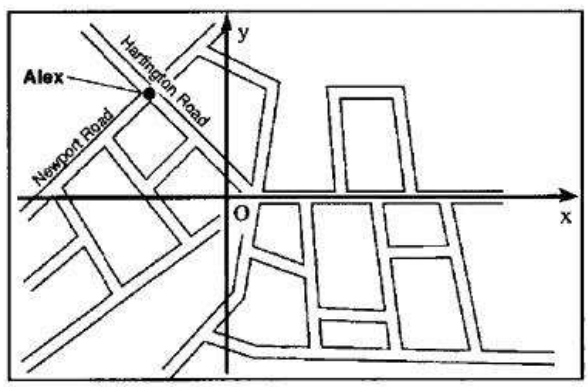
(*a*) Find the equations of the diagonals of this kite and the coordinates of the point where they intersect. **7**

(*b*) Find the coordinates of the fourth vertex S. **2**

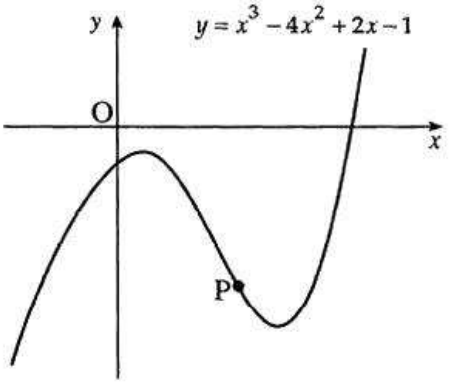
[SQA] 9. Relative to the axes shown and with an appropriate scale, Alex stands at the point (–2, 3) where Hartington Road meets Newport Road.

(*a*) Find the equation of Newport Road which is perpendicular to Hartington Road. **3**

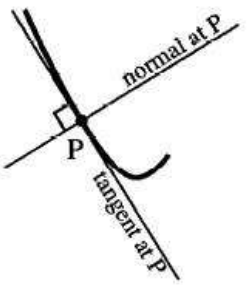
(*b*) Brenda is waiting for a bus at the point (–5, 1). Show that Brenda is standing on Newport Road. **1**



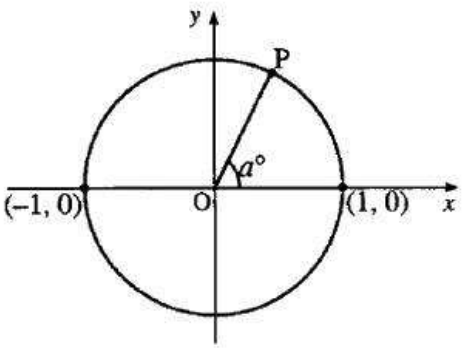
[SQA] 10. (*a*) The diagram shows an incomplete sketch of the curve with equation $y = x^3 - 4x^2 + 2x - 1$. Find the equation of the tangent to the curve at the point P where $x = 2$. **(5)**

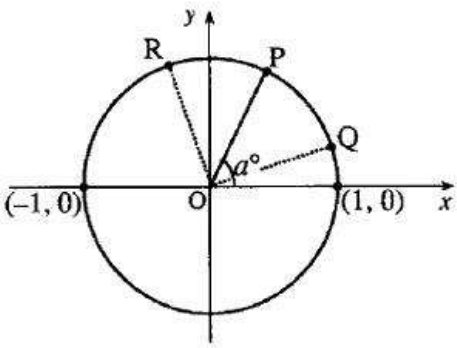


(*b*) The normal to the curve at P is defined as the straight line through P which is perpendicular to the tangent to the curve at P.
Find the angle which the normal at P makes with the positive direction of the x-axis. **(2)**



[SQA] 11. The diagram shows a circle of radius 1 unit and centre the origin. The radius OP makes an angle $a°$ with the positive direction of the x-axis.





(*a*) Show that P is the point $(\cos a°, \sin a°)$. 1

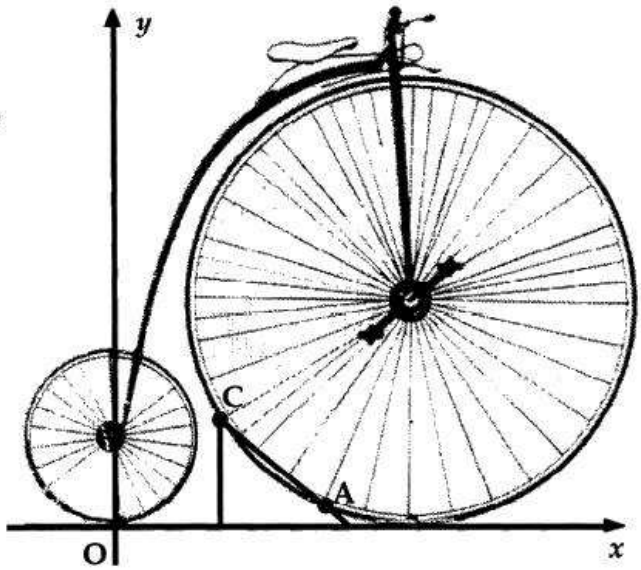
(*b*) If $P\hat{O}Q = 45°$, deduce the coordinates of Q in terms of a. 1

(*c*) If $P\hat{O}R = 45°$, deduce the coordinates of R in terms of a. 1

(*d*) Hence find an expression for the gradient of QR in its simplest form. 4

(*e*) Show that the tangent to the circle at P is parallel to QR. 2

[SQA] 12. A penny-farthing bicycle on display in a museum is supported by a stand at points A and C. A and C lie on the front wheel.
With coordinate axes as shown and 1 unit = 5cm, the equation of the rear wheel (the small wheel) is $x^2 + y^2 - 6y = 0$ and
the equation of the front wheel is $x^2 + y^2 - 28x - 20y + 196 = 0$.

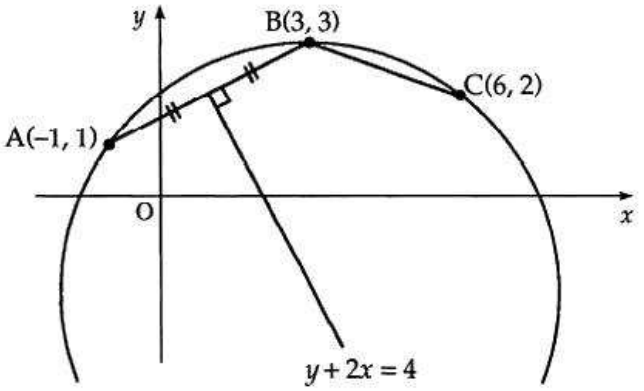


(*a*) (i) Find the distance between the centres of the two wheels.
(ii) Hence calculate the clearance, i.e. the smallest gap, between the front and rear wheels. Give your answer to the nearest millimetre. **(8)**

(*b*) B(7,3) is half-way between A and C, and P is the centre of the front wheel.
(i) Find the gradient of PB.
(ii) Hence find the equation of AC and the coordinates of A and C. **(8)**

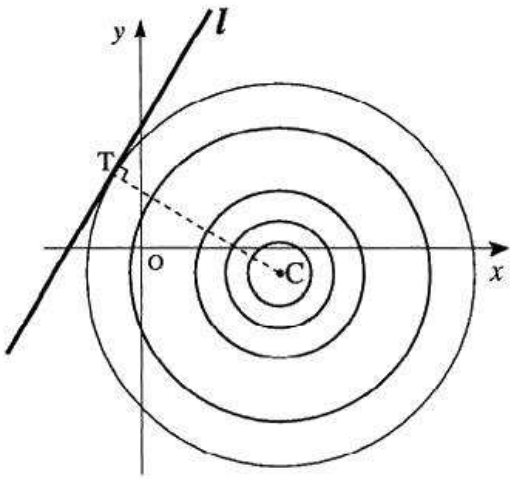
[SQA] 13. Find the equation of the tangent at the point $(3, 4)$ on the circle $x^2 + y^2 + 2x - 4y - 15 = 0$. **4**

[SQA] 14. (*a*) In the diagram, A is the point (–1, 1), B is (3, 3) and C is (6, 2). The perpendicular bisector of AB has equation $y + 2x = 4$. Find the equation of the perpendicular bisector of BC. **(4)**



(*b*) Find the centre and the equation of the circle which passes through A, B and C. **(6)**

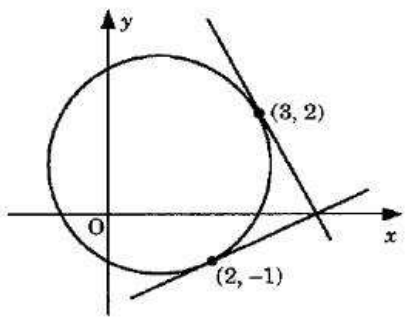
[SQA] 15. In an experiment with a ripple tank, a series of concentric circles with centre C(4,–1) is formed as shown in the diagram.
The line l with equation $y = 2x + 6$ represents a barrier placed in the tank.
The largest complete circle touches the barrier at the point T.



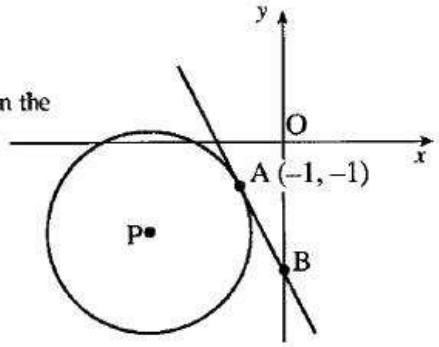
(*a*) Find the equation of the radius CT. **(3)**

(*b*) Find the equation of the largest complete circle. **(5)**

[SQA] 16. The circle shown in the diagram has equation
$(x-1)^2 + (y-1)^2 = 5$.
Tangents are drawn at the points (3, 2) and (2, –1).
Write down the coordinates of the centre of the circle and hence show that the tangents are perpendicular to each other. 4

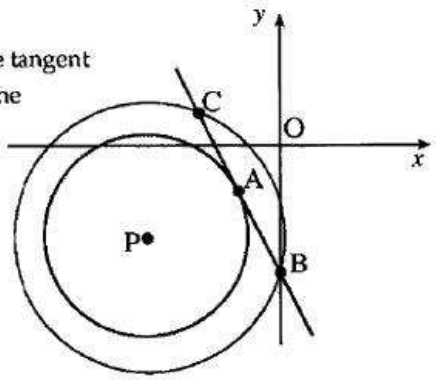


[SQA] 17. (*a*) The diagram shows a circle, centre P, with equation $x^2 + y^2 + 6x + 4y + 8 = 0$.
Find the equation of the tangent at the point A(–1, –1) on the circle. **4**



(*b*) The tangent crosses the *y*-axis at B.
Find the coordinates of B. **1**

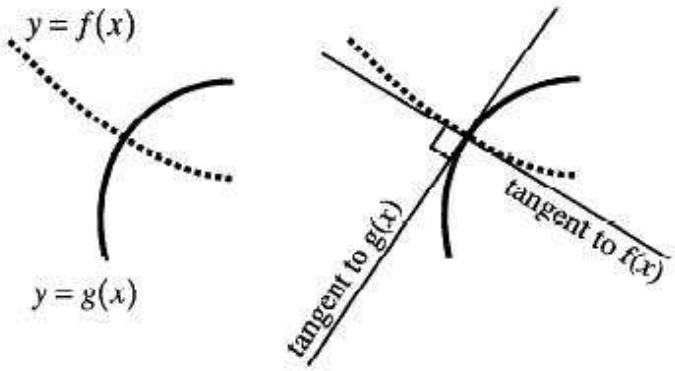
(*c*) Another circle, centre P, is drawn passing through B. The tangent at A meets the second circle at the point C, as shown in the diagram.
Write down the coordinates of C. **1**



(*d*) Find the equation of the circle with BC as diameter. **2**

[SQA] 18.

Two curves, $y = f(x)$ and $y = g(x)$, are called orthogonal if, at each point of intersection, their tangents are at right angles to each other.



(*a*) Diagram 1 shows the parabola with equation $y = 6 + \frac{1}{9}x^2$ and the circle M with equation $x^2 + (y-5)^2 = 13$.
These two curves intersect at (3, 7) and (–3, 7).
Prove that these curves are orthogonal. **(6)**

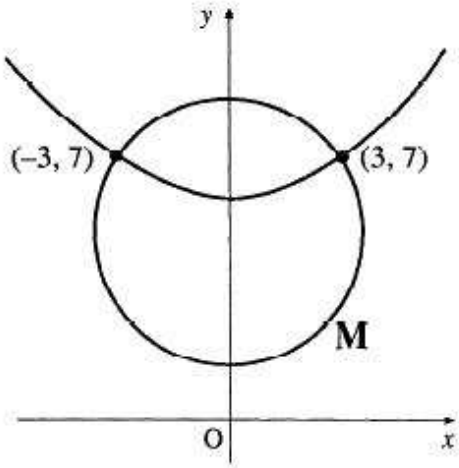


Diagram 1

(*b*) Diagram 2 shows the circle M, from (*a*) above, which is orthogonal to the circle N. The circles intersect at (3, 7) and (–3, 7).

(i) Write down the equation of the tangent to circle M at the point (–3, 7). **(1)**

(ii) Hence find the equation of circle N. **(3)**

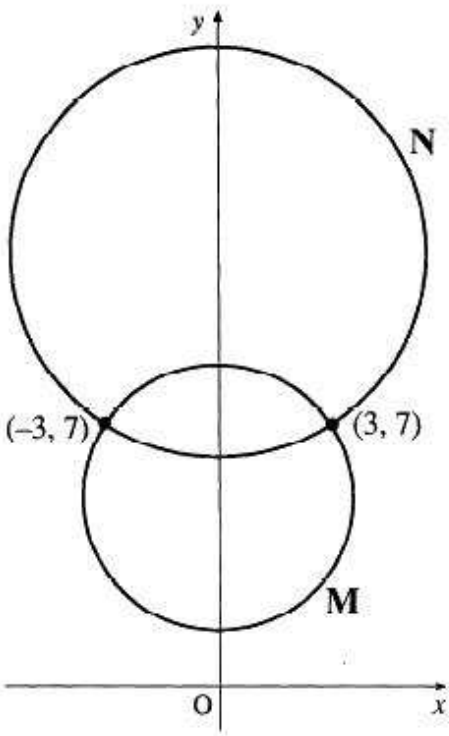


Diagram 2